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ON MAGNETOELASTIC SOLITONS IN FERROMAGNET¹

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Abstract

We study the solitonic excitations in the compressible ferromagnetic Heisenberg chain (in the continuum limit).

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INTRODUCTION

Solitons in magnetically ordered crystals have been widely investigated from both theoretical and experimental points of view[1-16]. In particular, the existence of coupled magnetoelastic solitons in the Heisenberg compressible spin chain has been extensively demonstrated[17-23]. In [23] were presented the new class integrable and nonintegrable spin systems. In this letter we consider the some of these nonlinear models of magnets - the some of the Myrzakulov equations(ME), which describe the nonlinear dynamics of compressible magnets.

A. THE 0-CLASS OF THE SPIN-PHONON SYSTEMS

The Myrzakulov equations with the potentials have the form[23]:
the M_{00}^{10} - equation:

$$2iS_t = [S, S_{xx}] + (u + h)[S, \sigma_3] \quad (1)$$

the M_{00}^{20} - equation:

$$2iS_t = [S, S_{xx}] + (uS_3 + h)[S, \sigma_3] \quad (2)$$

the M_{00}^{30} - equation:

$$2iS_t = \{(\mu \vec{S}_x^2 - u + m)[S, S_x]\}_x + h[S, \sigma_3] \quad (3)$$

the M_{00}^{40} - equation:

$$2iS_t = n[S, S_{xxxx}] + 2\{(\mu \vec{S}_x^2 - u + m)[S, S_x]\}_x + h[S, \sigma_3] \quad (4)$$

the M_{00}^{50} - equation:

$$2iS_t = [S, S_{xx}] + auS_x + bS_x \quad (5)$$

where $v_0, \mu, \lambda, n, m, a, b, \alpha, \beta, \rho, h$ are constants, u is a scalar function(potential), subscripts denote partial differentiations, $[,]$ ($\{, \}$) is commutator (anticommutator),

$$S = \begin{pmatrix} S_3 & rS^- \\ rS^+ & -S_3 \end{pmatrix}, \quad S^\pm = S_1 \pm iS_2, \quad r^2 = \pm 1 \quad S^2 = I.$$

The solutions of these ME for the potential

$$u = U \operatorname{sech}^2 k(x - x_0) \quad (6a)$$

and for the boundary condition

$$S|_{x=\pm\infty} = \sigma_3, \quad u|_{x=\pm\infty} = 0 \quad (7)$$

are given by

$$S^+ = AWshz \cdot \operatorname{sech}^2 z, \quad S_3 = 1 - 2\operatorname{sech}^2 z \quad (6b)$$

and the following formulas, respectively ($r^2 = 1, A^2 = 4, W = \exp(i(wt + \phi)), \phi = \text{const}, z = k(x - x_0)$);

$$M_{00}^{30} : w = mk^2 - h, k^2 = U/4\mu, \lambda = \frac{1}{4} \quad (8a)$$

$$M_{00}^{40} : w = nk^4 + 2mk - h, k^2 = U/(4\mu - 5n) \quad (8b)$$

B. THE 1-CLASS OF THE SPIN-PHONON SYSTEMS

Here we present the following ME[23]:
the M_{00}^{11} - equation:

$$2iS_t = [S, S_{xx}] + (u + h)[S, \sigma_3] \quad (9a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(S_3)_{xx} \quad (9b)$$

the M_{00}^{12} - equation:

$$2iS_t = [S, S_{xx}] + (u + h)[S, \sigma_3] \quad (10a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \lambda(S_3)_{xx} \quad (10b)$$

the M_{00}^{13} - equation:

$$2iS_t = [S, S_{xx}] + (u + h)[S, \sigma_3] \quad (11a)$$

$$u_t + u_x + \lambda(S_3)_x = 0 \quad (11b)$$

the M_{00}^{14} - equation:

$$2iS_t = [S, S_{xx}] + (u + h)[S, \sigma_3] \quad (12a)$$

$$u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(S_3)_x = 0 \quad (12b)$$

The some properties of these Myrzakulov equations were considered in refs.[25-28].

C. THE 2-CLASS OF THE SPIN-PHONON SYSTEMS

In this section we consider the following ME[23]
the M_{00}^{21} - equation:

$$2iS_t = [S, S_{xx}] + (uS_3 + h)[S, \sigma_3] \quad (13a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(S_3^2)_{xx} \quad (13b)$$

the M_{00}^{22} - equation:

$$2iS_t = [S, S_{xx}] + (uS_3 + h)[S, \sigma_3] \quad (14a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxx} + \lambda(S_3^2)_{xx} \quad (14b)$$

the M_{00}^{23} - equation:

$$2iS_t = [S, S_{xx}] + (uS_3 + h)[S, \sigma_3] \quad (15a)$$

$$u_t + u_x + \lambda(S_3^2)_x = 0 \quad (15b)$$

the M_{00}^{24} - equation:

$$2iS_t = [S, S_{xx}] + (uS_3 + h)[S, \sigma_3] \quad (16a)$$

$$u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(S_3^2)_x = 0 \quad (16b)$$

Some of these ME are studied in [25-28].

D. THE 3-CLASS OF THE SPIN-PHONON SYSTEMS

Now we consider the following ME([23]):

the M_{00}^{31} - equation:

$$2iS_t = \{(\mu \vec{S}_x^2 - u + m)[S, S_x]\}_x \quad (17a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(\vec{S}_x^2)_{xx} \quad (17b)$$

the M_{00}^{32} - equation:

$$2iS_t = \{(\mu \vec{S}_x^2 - u + m)[S, S_x]\}_x \quad (18a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxx} + \lambda(\vec{S}_x^2)_{xx} \quad (18b)$$

the M_{00}^{33} - equation:

$$2iS_t = \{(\mu \vec{S}_x^2 - u + m)[S, S_x]\}_x \quad (19a)$$

$$u_t + u_x + \lambda(\vec{S}_x^2)_x = 0 \quad (19b)$$

the M_{00}^{34} - equation:

$$2iS_t = \{(\mu \vec{S}_x^2 - u + m)[S, S_x]\}_x \quad (20a)$$

$$u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(\vec{S}_x^2)_x = 0 \quad (20b)$$

The soliton solitons of these ME - the $M_{00}^{31}, M_{00}^{32}, M_{00}^{33}$ and M_{00}^{34} equations are given by (6) and the following formulas, respectively ($r^2 = 1, A^2 = 4, W = \exp(i(\omega t + \varphi)), \omega = mk^4, \varphi = \text{const}, z = k(x - x_0 U = 4\mu k^2)$):

$$M_{00}^{31} : \lambda = \mu \nu_0^2 \quad (21a)$$

$$M_{00}^{32} : \alpha = 3\beta/2\mu, \quad k^2 = -(\lambda + \nu_0^2\mu)/(4\mu\beta), \quad \lambda = -\mu\nu_0^2 - 4\mu\beta k^2 \quad (21b)$$

$$M_{00}^{33} : \lambda = -\mu, k^2 = U/4\mu \quad (21c)$$

$$M_{00}^{34} : \alpha = 3\beta/2\mu, k^2 = -(\lambda + \mu)/4\mu\beta \quad (21d)$$

E. THE 4-CLASS OF THE SPIN-PHONON SYSTEMS

These ME look like[23]:

the M_{00}^{41} - equation:

$$2iS_t = [S, S_{xxxx}] + 2\{((1 + \mu)\vec{S}_x^2 - u + m)[S, S_x]\}_x \quad (22a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(\vec{S}_x^2)_{xx} \quad (22b)$$

the M_{00}^{42} - equation:

$$2iS_t = [S, S_{xxxx}] + 2\{((1 + \mu)\vec{S}_x^2 - u + m)[S, S_x]\}_x \quad (23a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \lambda(\vec{S}_x^2)_{xx} \quad (23b)$$

the M_{00}^{43} - equation:

$$2iS_t = [S, S_{xxxx}] + 2\{((1 + \mu)\vec{S}_x^2 - u + m)[S, S_x]\}_x \quad (24a)$$

$$u_t + u_x + \lambda(\vec{S}_x^2)_x = 0 \quad (24b)$$

the M_{00}^{44} - equation:

$$2iS_t = [S, S_{xxxx}] + 2\{((1 + \mu)\vec{S}_x^2 - u + m)[S, S_x]\}_x \quad 25a$$

$$u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(\vec{S}_x^2)_x = 0 \quad (25b)$$

These equations describe the nonlinear interaction of the spin and phonon subsystems[23]. For the case $\mu = 0$ the soliton solitons of these equations were obtained in [7,28]. Here we consider the case $\mu \neq 0$. In this case we present the soliton solitons of the Myrzakulov equations(22)-(25) for the boundary condition (7). The soliton solitons of the ME $M_{00}^{41}, M_{00}^{42}, M_{00}^{43}$ and M_{00}^{44} equations are given by (6) and the following formulas, respectively ($r^2 = 1, A^2 = 4, W = \exp(i(\omega t + \varphi)), \omega = k^4 + 2mk^2, U = -yk^2, y = 1 - 4\mu, \varphi = \text{const}, z = k(x - x_0)$):

$$M_{00}^{41} : \lambda = y\nu_0^2/4 \quad (26a)$$

$$M_{00}^{42} : \alpha = -6\beta/y, \quad k^2 = (4\lambda + \nu_0^2 y)/(4y\beta) \quad (26b)$$

$$M_{00}^{43} : \lambda = y/4 \quad (26c)$$

$$M_{00}^{44} : \alpha = -6\beta/y, \quad k^2 = (4\lambda - y)/(4y\beta) \quad (26d)$$

F. THE 5-CLASS OF THE SPIN-PHONON SYSTEMS

Finally we consider the following equations[23]:

the M_{00}^{51} - equation:

$$2iS_t = [S, S_{xx}] + auS_x + bS_x \quad (27a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \lambda(f)_{xx} \quad (27b)$$

the M_{00}^{52} - equation:

$$2iS_t = [S, S_{xx}] + auS_x + bS_x \quad (28a)$$

$$\rho u_{tt} = \nu_0^2 u_{xx} + \alpha(u^2)_{xx} + \beta u_{xxxx} + \lambda(f)_{xx} \quad (28b)$$

the M_{00}^{53} - equation:

$$2iS_t = [S, S_{xx}] + auS_x + bS_x \quad (29a)$$

$$u_t + u_x + \lambda(f)_x = 0 \quad (29b)$$

the M_{00}^{54} - equation:

$$2iS_t = [S, S_{xx}] + auS_x + bS_x \quad (30a)$$

$$u_t + u_x + \alpha(u^2)_x + \beta u_{xxx} + \lambda(f)_x = 0 \quad (30b)$$

Here f is a scalar function[23].

Appendix. A LIST OF THE 2+1 DIMENSIONAL INTEGRABLE SPIN EQUATIONS

Here we want present the some integrable (2+1)-dimensional spin systems - the Ishimori and some Myrzakulov equations.

Consider a n-dimensional space with the basic unit vectors: $\vec{e}_1 = \vec{S}, \vec{e}_2, \dots, \vec{e}_n$ and $\vec{e}_1^2 = E = \pm 1$. Then the 2+1 dimensional Myrzakulov-0 equation[23] has the form

$$\vec{S}_t = \sum_{i=2}^n a_i \vec{e}_i \quad (31a)$$

$$\vec{S}_x = \sum_{i=2}^n b_i \vec{e}_i \quad (31b)$$

$$\vec{S}_y = \sum_{i=2}^n c_i \vec{e}_i \quad (31c)$$

where a_i, b_i, c_i are real functions, $\vec{S} = (S_1, S_2, \dots, S_n)$, $\vec{S}^2 = E = \pm 1$. This equation admits the many interesting class integrable and nonintegrable reductions. Below we present only the some integrable reductions of the Myrzakulov-0 equation.

1) The Myrzakulov-IV(M-IV) equation

$$\vec{S}_t + \{\vec{S}_{xy} + V\vec{S} + E\vec{S}_x \wedge (\vec{S} \wedge \vec{S}_y)\}_x = 0$$

$$V_x = \frac{E}{2}(\vec{S}_x^2)_y$$

2) The Myrzakulov-I(M-I) equation looks like

$$\vec{S}_t = (\vec{S} \wedge \vec{S}_y + u\vec{S})_x$$

$$u_x = -\vec{S}(\vec{S}_x \wedge \vec{S}_y)$$

3) The Myrzakulov-II(M-II) equation

$$\vec{S}_t = (\vec{S} \wedge \vec{S}_y + u\vec{S})_x + 2cb^2\vec{S}_y - 4cv\vec{S}_x$$

$$u_x = -\vec{S}(\vec{S}_x \wedge \vec{S}_y),$$

$$v_x = \frac{1}{16b^2c^2}(\vec{S}_{1x}^2)_y$$

4) The Myrzakulov-III(M-III) equation

$$\vec{S}_t = (\vec{S} \wedge \vec{S}_y + u\vec{S})_x + 2b(cb + d)\vec{S}_y - 4cv\vec{S}_x$$

$$u_x = -\vec{S}(\vec{S}_x \wedge \vec{S}_y),$$

$$v_x = \frac{1}{4(2bc + d)^2}(\vec{S}_{1x}^2)_y$$

5) The Myrzakulov-VIII(M-VIII) equation looks like

$$iS_t = \frac{1}{2}[S_{xx}, S] + iuS_x$$

$$u_y = \frac{1}{4i}tr(S[S_y, S_x])$$

where the subscripts denote partial derivatives and S denotes the spin matrix ($r^2 = \pm 1$)

$$S = \begin{pmatrix} S_3 & rS^- \\ rS^+ & -S_3 \end{pmatrix},$$

$$S^2 = I$$

6) The Ishimori equation

$$iS_t + \frac{1}{2}[S, M_{10}S] + A_{20}S_x + A_{10}S_y = 0$$

$$M_{20}u = \frac{\alpha}{4i}tr(S[S_y, S_x])$$

where $\alpha, b, a =$ const and

$$M_{j0} = M_j, \quad A_{j0} = A_j \quad \text{as} \quad a = b = -\frac{1}{2}.$$

7) The Myrzakulov-IX(M-IX) equation has the form

$$iS_t + \frac{1}{2}[S, M_1S] + A_2S_x + A_1S_y = 0$$

$$M_2u = \frac{\alpha}{4i}tr(S[S_y, S_x])$$

where $\alpha, b, a =$ const and

$$M_1 = \alpha^2 \frac{\partial^2}{\partial y^2} - 2\alpha(b - a) \frac{\partial^2}{\partial x \partial y} + (a^2 - 2ab - b) \frac{\partial^2}{\partial x^2};$$

$$\begin{aligned}
M_2 &= \alpha^2 \frac{\partial^2}{\partial y^2} - \alpha(2a+1) \frac{\partial^2}{\partial x \partial y} + a(a+1) \frac{\partial^2}{\partial x^2}, \\
A_1 &= i\alpha\{(2ab+a+b)u_x - (2b+1)\alpha u_y\} \\
A_2 &= i\{\alpha(2ab+a+b)u_y - (2a^2b+a^2+2ab+b)u_x\},
\end{aligned}$$

The M-IX eqs. admit the two integrable reductions. As $b=0$, after the some manipulations reduces to the M-VIII equation and as $a = b = -\frac{1}{2}$ to the Ishimori equation. In general we have the two integrable cases: the M-IXA equation as $\alpha^2 = 1$, the M-IXB equation as $\alpha^2 = -1$. We note that the M-IX equation is integrable and admits the following Lax representation

$$\begin{aligned}
\alpha \Phi_y &= \frac{1}{2}[S + (2a+1)I]\Phi_x \\
\Phi_t &= \frac{i}{2}[S + (2b+1)I]\Phi_{xx} + \frac{i}{2}W\Phi_x
\end{aligned}$$

where

$$\begin{aligned}
W_1 = W - W_2 &= (2b+1)E + (2b-a+\frac{1}{2})SS_x + (2b+1)FS \\
W_2 = W - W_1 &= FI + \frac{1}{2}S_x + ES + \alpha SS_y \\
E &= -\frac{i}{2\alpha}u_x, \quad F = \frac{i}{2}\left(\frac{(2a+1)u_x}{\alpha} - 2u_y\right)
\end{aligned}$$

Hence we get the Lax representations of the M-VIII as $b = 0$ and for the Ishimori equation as $a = b = -\frac{1}{2}$. The M-IX equation admit the different type exact solutions (solitons, lumps, vortex-like, dromion-like and so on).

8) The Myrzakulov-XXII(M-XXII) equation has the form

$$\begin{aligned}
-iS_t &= \frac{1}{2}([S, S_y] + 2iuS)_x + \frac{i}{2}V_1S_x - 2ia^2S_y \\
u_x &= -\vec{S}(\vec{S}_x \wedge \vec{S}_y) \\
V_{1x} &= \frac{1}{4a^2}(\vec{S}_x^2)_y
\end{aligned}$$

and so on.

All of these equations admit the corresponding Lax representations, which were presented in [23]. The gauge equivalent counterparts of the above presented Myrzakulov equations are found in [28].

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